

# Dense uniform hypergraphs have long Berge cycles

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Well-known bounds on the number of edges in  $n$ -vertex graphs that have no long paths or no long cycles were given by Erdős and Gallai in 1959. An analog of the Erdős–Gallai Path Theorem for Berge paths of length  $k$  in  $n$ -vertex  $r$ -uniform hypergraphs for  $k \neq r + 1$  was recently proved by Győri, Katona and Lemons. Then Davoodi, Győri, Methuku and Tompkins settled the remaining case  $k = r + 1$ . Their bounds are exact for every  $k$  and  $r$  for infinitely many  $n$ . They also proved a better bound for 2-connected hypergraphs when  $n \gg k \gg r$ .

We prove an analog of the Erdős–Gallai Cycle Theorem for Berge cycles of length at least  $k$  in  $n$ -vertex  $r$ -uniform hypergraphs for  $k \notin \{r + 1, r + 2\}$ . We also describe the hypergraphs for which our bounds are exact. Our results imply somewhat refined versions of the above mentioned theorems for Berge paths when  $k \notin \{r + 1, r + 2\}$ . We also give a better bound for 2-connected hypergraphs when  $n \gg k > 8r$ .

This is joint work with Z. Füredi and R. Luo.